GOVERNMENT ENGINEERING COLLEGE, GANDHINAGAR
B.E. Sem I (All Branches) A.Y.2021-22
Tutorial-1

Subject:- Mathematics-1(3110014)

Topic:- Indeterminate Form, Improper Integral, Convergence &Divergence of the Integral, Beta & Gamma Function, Application of Definite Integral – Volume Using Cross section, Length of Plane Curves and Area of Surface of Revolution.

Weightage: 15%

(1)Evaluate following:

a) $\lim_{x\to 0}\frac{e^x+e^{-x}-x^2-2}{sin^2x-x^2}$

b) $\lim_{x\to 1}\frac{x^x-x}{x-1-logx}$

c) $\lim_{x\to y}\frac{x^y-y^x}{x^x-y^y}$

d) $\lim_{x\to 0}\frac{\log(sinx)}{\log(tanx)}$

e) $\lim_{x\to 1}\left[\frac{1}{logx}-\frac{x}{x-1}\right]$

f) $\lim_{x\to 0}\left[\frac{1}{x^2}-\frac{1}{sin^2x}\right]$

g) $\lim_{x\to 0}x^n logx$

h) $\lim_{x\to 1}(1-x)tan\frac{\pi x}{2}$

i) $\lim_{x\to 0}\left[\frac{tanx}{x}\right]^{1/x}$

j) $\lim_{x\to 1}[x-1]^{x-1}$

k) $\lim_{x\to {\pi}/{2}}[secx]^{cotx}$

l) $\lim_{x\to 0}\left[\frac{a^x+b^x+c^x}{3}\right]^{1/x}$.

(2) Evaluate. (i) $\int_5^\infty\frac{dx}{x^3}$ (ii) $\int_{-\infty}^{-2}e^x dx.$

(3) Check the convergence of $\int_5^\infty\frac{7x+4}{x^2+9}dx.$

(4) Is $\int_4^\infty\frac{sin^2x}{\sqrt{x}(x-1)}dx$ convergent?

(5) Evaluate$\int_0^\infty\frac{dx}{x^2}$.

(6) Prove that $B(m,n)=B(m+1,n)+B(m,n+1)$ & $\int_0^1\frac{x}{\sqrt{1-x^5}}dx=\frac{1}{5}B(\frac{2}{5},\frac{1}{5})$

(7) Prove that$\int_0^\infty\sqrt{x}\,e^{-x^2}dx\times\int_0^\infty\frac{e^{-x^2}}{\sqrt{x}}dx=\frac{\pi}{2\sqrt{2}}$.

(8) Evaluate following:

(i) $\int_0^\infty e^{-x^2}dx$ (ii) $\int_0^1 y^{q-1}(log\frac{1}{y})^{p-1}dy$ (iii) $\int_0^1(xlogx)^3dx$

(iv) $\int_3^7\sqrt[4]{(x-3)(7-x)}dx$ (v) $\int_0^{\pi/2}sin^3xcos^{5/2}x\,dx$ (vi)$\int_{-1}^1(1-x^2)^n dx$

(vii)$\int_0^{\pi/2}\frac{d\theta}{\sqrt{sin\theta}}\times\int_0^{\pi/2}\sqrt{sin\theta}\,d\theta.$

(9) Using slicing method find the volume of solid ball of radius a.

(10) Find the entire length of the cardioid $r=a(1+cos\theta)$.

(11) Find the length in first quadrant of the curve $x=a\,cos^3t, y=a\,sin^3t.$

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(12) Find the surface area of the solid generated by revolution of $r = 2a\ cos\theta$ about the initial line

Tutorial-2

Subject:- Mathematics-1 **Topic:- Convergence of Sequence and Series.**

Weightage: 20%

1. Test the convergence of the following sequences.

   (1) $\frac{2n^2+n}{n^2+1}$ (2) $\frac{n!}{n^n}$ (3) $a_n = \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \ldots\ldots\ldots + \frac{1}{n!}$ (4) $\left(\frac{n+1}{n-1}\right)^n$

2. Find the sum of the following series

   (1) $\sum_{n=1}^{\infty} \frac{1}{(2n-1)(2n+1)}$ (2) $\frac{2^{3n}}{3^{2n}}$ (3) $1 + \frac{2}{3} + \frac{4}{9} + \frac{8}{27} + \frac{16}{81} + \cdots$

3. Test the convergence of following series.

   (1) $\sum_{n=1}^{\infty} \frac{1}{n^p}$ (p-test) (2) $\sum_{n=1}^{\infty} sin\frac{1}{n}$ (3) $\sum_{n=1}^{\infty} \frac{n!(2)^n}{n^n}$ (4) $\sum_{n=1}^{\infty} \frac{1}{3^n+1}$

   (5) $\sum_{n=1}^{\infty} \left(\frac{n}{2n+5}\right)^n$ (6) $\sum_{n=1}^{\infty} \frac{1}{2} - \frac{2}{5} + \frac{3}{10} - \frac{4}{17} + \cdots$ (7) $\sum_{n=1}^{\infty} (\sqrt{n^2+1} - n)$

   (8) $\sum_{n=1}^{\infty} \left(\frac{n^2}{2^n} + \frac{1}{n^2}\right)$ (9) $\sum_{n=1}^{\infty} cot^{-1} n$ (10) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$

4. For what values of x does the following series converge? And find their radius of convergence.

   (1) $\frac{1}{1.2.3} + \frac{x}{4.5.6} + \frac{(x)^2}{7.8.9} + \ldots..$ (2) $x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots$

5. Find Taylor series for following functions at given point [OR in powers of (x-a) .

   (1) $f(x) = \sqrt{x}; a = 25$ (2) $f(x) = \cos x\ ; a = \frac{\pi}{4}$ (3) $f(x) = x \cdot \cos x\ ; a = 0$

   (4) $f(x) = e^x\ ; a = 1$ (5) $f(x) = \frac{1}{x}\ ; a = -2$. (6) $3x^3 + 7x^2 - 5x - 4$ at x = 0

6. Find Maclaurin series for following functions.

   (1) $f(x) = \cosh x$ (2) $f(x) = sin^{-1}x$ (3) $f(x) = e^{-2x}$ (4) $f(x) = tan^{-1}x$.

Tutorial-3

Subject:- Mathematics-1 **Topic:- Fourier Series**

Weightage: 10%

Ex-1 Find Fourier series of the function of $f(x) = x^2 - 2\ , -2 \le x \le 2\ ,$ p= 2L=4.

Ex-2 Find Fourier series of the function of $f(x) = \left(\frac{\pi - x}{2}\right)^2$ $, 0 \le x \le 2\pi$

Ex-3 Find Fourier series of the function $f(x)$ which is assumed to have the period 2π where

$f(x) = x + |x|\ ;\ -\pi < x < \pi.$

Ex-4 Find Fourier series of the function of $f(x) = xcosx,\ (-\pi < x < \pi).$

Ex-5 Find Fourier series of the function of

(1) $f(x) = \begin{cases} x, & 0 \le x \le 2 \\ 4 - x, & 2 \le x \le 4, \end{cases}$ (2) $f(x) = \begin{cases} \pi + x, & -\pi < x < 0 \\ \pi - x, & 0 < x < \pi \end{cases}$

Ex-6 Find Fourier series of the function of $f(x) = \begin{cases} -\pi, & -\pi < x < 0 \\ x - \pi, & 0 < x < \pi \end{cases}$

Ex-7 Find Fourier series of $f(x) = \begin{cases} k, -\pi < x < 0 \\ -k, \quad 0 < x < \pi, \end{cases}$ hence deduct that

$\frac{1}{1} - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots = \frac{\pi}{4}.$

Ex-8 Prove that

$$\frac{l}{2} - x = \frac{l}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin\frac{2n\pi x}{l}, \qquad 0 < x < l.$$

Ex-9 Find Fourier sine series of the function

(1) $f(x) = e^x;\ 0 < x < l.$
(2) $f(x) = \pi - x;\ 0 < x < \pi$
(3) $f(x) = x;\ 0 < x < 2.$
(4) $f(x) = cos\,2x\,, 0 \le x \le \pi$

Ex-10 Find Fourier cosine series of the function

(1) $f(x) = e^x;\ 0 < x < \pi.$
(2) $f(x) = x;\ 0 < x < 2.$
(3) $f(x) = (x-1)^2;\ 0 < x < 1.$

Tutorial-4

Subject:- Mathematics 01 **Topic:- Partial Differentiation**

Weightage: 20%

Ex-1 Find the limits:

(1) $\lim_{(x,y)\to(5,-2)} (x^4 + 4x^3y - 5xy^2)$

(2) $\lim_{\substack{(x,y)\to(0,0)\\ x\neq y}} \frac{x-y+2\sqrt{x}-2\sqrt{y}}{\sqrt{x}-\sqrt{y}}$

(3) $\lim_{(x,y)\to(e,1)} \ln\left(\frac{x}{y^2}\right)$

(4) $\lim_{(x,y)\to(\frac{\pi}{2},0)} \frac{\cos y+1}{y-\sin x}$

Ex-2 Determine whether limit of the following functions are exit or not about $(x,y)\to(0,0)$? If they exist find the value of the limit.

(1) $f(x,y) = \frac{x^3 - xy^2}{x^2+y^2}$ (2) $f(x,y) = \frac{y^2}{x^2+y^2}$ (3) $f(x,y) = \frac{x^2y}{x^4+y^2}$

Ex-3 Show that $\lim_{(x,y)\to(0,0)} \frac{x^2y}{x^4+y^2} = 0.$

Ex-4 Discuss the continuity of $f(x,y) = \begin{cases} \frac{2xy}{x^2+y^2} & ; \quad (x,y)\neq(0,0) \\ 0 & ; \quad (x,y) = (0,0) \end{cases}.$

Ex-5 If $z = x + y^x$, prove that $\frac{\partial^2 z}{\partial x\,\partial y} = \frac{\partial^2 z}{\partial y\,\partial x}$.

Ex-6 For $u = tan^{-1}(y/x)$, prove the following (1) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ (2) $\frac{\partial^2 u}{\partial x\,\partial y} = \frac{\partial^2 u}{\partial y\,\partial x}$

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Ex-7 If resistors of R_1 , R_2 and R_3 ohms are connected in parallel to make an R-ohm resistor, the value of Rcan be found from the equation $\frac{1}{R}=\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}$, find the value of $\frac{\partial R}{\partial R_2}$ when $R_1 = 30,\ R_2 = 45$ and $R_3 = 90$ ohms.

Ex-8 If $u = \ln(x^3 + y^3 - x^2y - xy^2)$ prove that $\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial y}\right)^2 u = -\frac{4}{(x+y)^2}$.

Ex-9 If $= e^{x/y} \sin\left(\frac{x}{y}\right) + e^{y/x} \cos\left(\frac{x}{y}\right)$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0$

Ex-10 If $u = \left(x^{1/5} + y^{1/5}\right)\left(x^{1/3} + y^{1/3}\right)$ show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} - \frac{8}{15}u = 0.$

Ex-11 If u $= cosec^{-1}\left(\frac{x^{\frac{1}{2}}+y^{\frac{1}{2}}}{x^{\frac{1}{3}}+y^{\frac{1}{3}}}\right)^{\frac{1}{2}}$ prove that $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\,\partial y} + y^2\frac{\partial^2 u}{\partial y^2} = \frac{1}{144}\tan u\,(13 + tan^2u).$

Ex-12 Evaluate $\frac{\partial W}{\partial u}$ and $\frac{\partial W}{\partial v}$ at point $(u, v) = (1, 2)$
for $W = xy + yz + zx, x = u + v, y = u - v, z = uv.$

Ex-13 If $u = f(x,y); x = e^s \cos t$ and $y = e^s \sin t$, show that $\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 = e^{-2s}\left[\left(\frac{\partial u}{\partial s}\right)^2 + \left(\frac{\partial u}{\partial t}\right)^2\right].$

Ex-14 If $u = f(x - y, y - z, z - x)$, show that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0.$

Ex-15 Find $\frac{du}{dt}$, (1) $u = y^2 - 4ax, x = at^2, y = 2at$(2) $u = sin^{-1}(x - y), x = 3t, y = 4t^3.$

Ex-16 Find the value of $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ using partial derivative $x^2 + y^3 = 7xy$

Ex-17 If $\emptyset = 3x^2y - y^3z^2$, find $grad\ \emptyset$ at the point (1, -2, -1).

Ex-18 Find the derivative of $\mathrm{f(x,y) = x^2 \sin 2y}$ at the point $(1, \mathrm{p}/2)$ in the direction of $\mathrm{v} = 3\hat{\imath} - 4\hat{\jmath}$.

Ex-19 Find the directional derivative of following functions at given point and direction:
(1) $f(x,y,z) = 2x^2 + 3y^2 + z^2$ at (2, 1, 3) in the direction $a = \hat{\imath} - 2\hat{k}$
(2) $\emptyset(x,y,z) = 4xz^3 - 3x^2y^2z$ at (2, -1, 2) in the direction $a = 2\hat{\imath} + 3\hat{\jmath} + 6\hat{k}$
(3) $g(x,y,z) = 3e^x\, cos(yz)$ at (0, 0, 0) in the direction $v = 2\hat{\imath} + \hat{\jmath} - 2\hat{k}$

Ex-20 Find the equation for the tangent plane and normal line at the point $\mathrm{P_0}$ on the given surface
(1) $x^2 + y^2 - 2xy - x + 3y - z = 4\,; P_0(2,-3,18)$
(2) $x^2 + y^2 + z^2 = 3;\ P_0(1,1,1)$ (3) $x^2 + y^2 - 4z = 5;\ P_0(3,4,5)$

Ex-21 Find the local extreme values and saddle points of the given functions
(1) $f(x,y) = xy - x^2 - y^2 - 2x - 2y + 4$ (3) $f(x,y) = xy$
(2) $f(x,y) = x^3 + 3xy + y^3$ (4) $f(x,y) = 4x^2 - 6xy + 5y^2 - 20x + 26y$

Ex-22 Find the absolute maxima and minima of the function $f(x,y) = 2 + 2x + 2y - x^2 - y^2$ on the triangular plate in the first quadrant bounded by the lines $x = 0, y = 0, y = 9 - x.$

Ex-23 Find the point on the plane $x + 2y + 3z = 13$ closet to the point (1, 1, 1).

Ex-24 Find the greatest and smallest values that the function $\mathrm{f(x,y) = xy}$ takes on the ellips $\frac{x^2}{8} + \frac{y^2}{2} = 1.$

Tutorial-5

Subject:- MATHEMATICS-1 **Topic:- MULTIPLE INTEGRALS.**

Weightage: 20%

1. Sketch the region of integration and evaluate the following integrals.
(a) $\int_0^3\int_0^2 (x^2 + y^2)dydx$ (b) $\int_0^1\int_0^{y^2} 3y^3e^{xy}dxdy$ (c)$\int_0^{\pi}\int_{x^2}^{x}(x^2 - y^2)dydx$
2. Integrate $f(x,y) = \frac{x}{y}$ over the region in the first quadrant bounded by the lines y=x, y=2x, x=1, x=2.
3. Integrate $f(x,y) = x^2 + y^2$ over the triangular region with vertices (0,0), (1,0) and (0,1).

4. Sketch the region of integration and evaluate the following integrals by reversing its order.
(a) $\int_0^{\pi} \int_x^{\pi} \frac{\sin y}{y} dydx$ (b)$\int_0^1 \int_y^1 x^2 \cdot e^{xy} dxdy$ (c) $\int_0^{2\sqrt{\ln 3}} \int_{\frac{y}{2}}^{\sqrt{\ln 3}} e^{x^2} dxdy$

5. Evaluate $\iint_R \; xdA$, where R is the region bounded by the square $y^2 = 4ax$ and $x^2 = 4ay$.

6. Evaluate $\iint_R \; xydA$, where R is the region bounded by the square x-axis, $x = 4$ and $x^2 = 16y$.

7. Evaluate $\iint_R \; x^2 dA$, where R is the region in the first quadrant bounded by the hyperbola $xy = 16$ and the lines $y = x, y = 0$ and $x = 8$.

8. Evaluate $\int_0^1 \int_{x^2}^{x} (x + y) dA$, by changing the order of integration.

9. Find the volume of the solid that is bounded above by the cylinder $z = x^2$ and below by the region enclosed by the parabola $y = 2 - x^2$ and the line $y = x$ in the XY-plane.

10. Evaluate the following integrals.
(a) $\int_0^1 \int_0^1 \int_0^1 (x^2 + y^2 + z^2) \, dzdydx$ (b)$\int_0^1 \int_0^{1-x} \int_0^{x+y} e^z \, dzdydx$
(c) $\int_0^{\frac{\pi}{4}} \int_0^{\ln \sec v} \int_{-\infty}^{2t} e^x \, dxdtdv$

11. Change the Cartesian integral into the polar integral then evaluate the polar integral.
(a) $\int_0^1 \int_0^{\sqrt{1-y^2}} (x^2 + y^2) dxdy$ (b)$\int_0^{\infty} \int_0^{\infty} e^{-(x^2+y^2)} dxdy$
(c) $\int_{-1}^1 \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} ln(x^2 + y^2 + 1) dxdy$

12. Evaluate $\int_0^4 \int_{x=\frac{y}{2}}^{x=\frac{y}{2}+1} \frac{2x - y}{2} dxdy$ by applying the transformation $u = \frac{2x - y}{2}, v = \frac{y}{2}$ and integrating over an appropriate region in the UV-plane.

13. (Applying transformation to integrate) Evaluate $\iint_R \; (x - y)^4 \cdot e^{x+y} \, dxdy$ where R is the square with vertices (1,0), (2,1),(1,2) and (0,1).

Tutorial-6

Subject:- MATHEMATICS-1 **Topic:-Linear algebra.**

Weightage: 15%

Ex-1 Find row echelon form for the following matrices.

(i) $\begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix}$ (ii) $\begin{bmatrix} 1 & 3 & 2 & 2 \\ 1 & 2 & 1 & 3 \\ 2 & 4 & 3 & 4 \\ 3 & 7 & 4 & 8 \end{bmatrix}$ (iii) $\begin{bmatrix} 0 & 3 & -6 \\ 3 & -7 & 8 \\ 3 & -9 & 12 \end{bmatrix}$

Ex-2 Find reduced row echelon form for the following matrices.

(i) $\begin{bmatrix} 5 & 10 & 2 \\ 1 & 2 & 3 \end{bmatrix}$ (ii) $\begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$ (iii) $\begin{bmatrix} 1 & 1 & 1 \\ 3 & 2 & -1 \\ 2 & 2 & 1 \end{bmatrix}$

Ex-3 Find the rank using row echelon form.

(i) $\begin{bmatrix} 0 & 1 & -3 & -1 \\ 0 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$ (ii) $\begin{bmatrix} 1 & 2 & -1 \\ -1 & 0 & 2 \\ 2 & 1 & -3 \end{bmatrix}$ (iii) $\begin{bmatrix} 2 & -1 & 3 & 1 \\ 1 & 4 & -2 & 1 \\ 5 & 2 & 4 & 3 \end{bmatrix}$

Ex-4 Using Gauss-Jordan method find A^{-1} if

(i) A= $\begin{bmatrix} 7 & 6 & 2 \\ -1 & 2 & 4 \\ 3 & 6 & 8 \end{bmatrix}$ (ii) A= $\begin{bmatrix} \frac{1}{5} & \frac{1}{5} & \frac{-2}{5} \\ \frac{1}{5} & \frac{1}{5} & \frac{1}{10} \\ \frac{1}{5} & \frac{-4}{5} & \frac{1}{10} \end{bmatrix}$

EX-5 Solve following system of equation using Gauss-elimination method.

(1)$4x + 2y + 3z = 4, 2x + 2y + z = 6, x + y + z = 0$

(2)$2x + 2y + 2z = 0, -2x + 5y + 2z = 1, 3x + y + 4z = -1$

Ex-6 Solve each of following system by Gauss-Jorden elimination Method.

(1)$2x - y + 3z + 4w = 9, x - 2z + 7w = 1, 3x - 3y + z + 5w = 8, 2x + y + 4z + 4w = 10$

(2) $-2y + 3z = 1, 3x + 6y - 3z = -2, 6x + 6y + 3z = 5$

Ex-7 Solve the following system of equations.

(1) $2x - y - 3z = 0, -x + 2y - 3z = 0, x + y + 4z = 0$

(2) $3x + y + z + w = 0, 5x - y + z - w = 0$

Ex-8 Find the values of $a \,\&\, b$ for which the equation
$x + 2y + 3z = 6, x + 3y + 5z = 9, 2x + 5y + az = b$
have (i) No Solution (ii)Unique Solution (iii) Infinite no. of solution.

Ex.9 Find the Eigen values and Eigen vectors of

(1) $\begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$ (2) $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$ (3) $\begin{bmatrix} -3 & -7 & -5 \\ 2 & 4 & 3 \\ 1 & 2 & 2 \end{bmatrix}$

Ex-10 Using Cayley – Hamilton theorem, find A^{-1}, $A = \begin{bmatrix} 7 & -1 & 3 \\ 6 & 1 & 4 \\ 2 & 4 & 8 \end{bmatrix}$

Ex-11 If A = $\begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$ then by using cayley -Hamilton theorem prove that,

$$A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I = \begin{bmatrix} 8 & 5 & 5 \\ 0 & 3 & 0 \\ 5 & 5 & 8 \end{bmatrix}$$

Ex-12 Find a matrix P that diagonalize the matrix A = $\begin{bmatrix} 1 & 0 \\ 6 & -1 \end{bmatrix}$ & determine$P^{-1}AP$.

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$$\text{Ans. } P = \begin{bmatrix} 0 & 1 \\ 1 & 3 \end{bmatrix}$$

Ex-13 Diagonalize the following matrix & obtain the modal matrix of $\begin{bmatrix} 9 & -1 & 9 \\ 3 & -1 & 3 \\ -7 & 1 & -7 \end{bmatrix}$

$$\text{Ans. } P = \begin{bmatrix} 1 & 1 & 4 \\ 0 & 1 & 1 \\ -1 & -1 & -3 \end{bmatrix}$$

Ex-14 Diagenalize the matrix $A = \begin{bmatrix} 3 & -2 & 4 \\ -2 & 6 & 2 \\ 4 & 2 & 3 \end{bmatrix}$ orthogonally Ans. $P = \begin{bmatrix} {}^{2}/_{3} & {}^{1}/_{\sqrt{5}} & \frac{4}{3\sqrt{5}} \\ {}^{1}/_{3} & {}^{-2}/_{\sqrt{5}} & {}^{2}/_{3\sqrt{5}} \\ {}^{-2}/_{3} & 0 & {}^{5}/_{3\sqrt{5}} \end{bmatrix}$

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GOVERNMENT ENGINEERING COLLEGE,GANDHINAGAR

B.E. Sem I (All Branches) A.Y. 2021-22

Assignment-1

Subject:- Mathematics-1(3110014)

Topic:- Indeterminate Form, Improper Integral, Convergence &Divergence of the Integral, Beta & Gamma Function, Application of Definite Integral – Volume Using Cross section, Length of Plane Curves and Area of Surface of Revolution.

Weightage: 15%

(1)Evaluate following:

(a) $\lim_{x\to 0}\frac{tan^2x-x^2}{x^2tan^2x}$ (b) $\lim_{x\to \pi/2}\frac{2x-\pi}{cosx}$ (c) $\lim_{x\to\infty}\frac{x^n}{e^x}$ (d) $\lim_{x\to 0}\frac{coshx-cosx}{\text{xsinx}}$

(e) $\lim_{x\to 0}\left[\frac{1}{x}-\frac{1}{e^x-1}\right]$ (f) $\lim_{x\to 0}\left[\frac{ae^x-bcosx+ce^{-x}}{xsinx}\right]=2\,, find\ the\ value\ of\ a,b,c.$ (g) $\lim_{x\to\infty}\frac{x[logx]^3}{1+x+x^2}$

(h) $\lim_{x\to\pi/4}(1-tanx)sec2x$ (i) $\lim_{x\to 0}\left[\frac{1}{x}\right]^{1-cosx}$ (j) $\lim_{x\to 0}[cosx]^{cotx}$ (k) $\lim_{x\to 1}[1-x^2]^{1/\log(1-x)}$ (l) $\lim_{x\to 0}[a^x-1]x.$

(2) Evaluate

(i) $\int_3^\infty\frac{dx}{x^{-2}}$ (ii) $\int_{-\infty}^0 xe^{-x^2}dx$ (iii) $\int_0^\infty\frac{1}{x^2+4}dx$

(3) Show that $\int_5^\infty e^{-x^2}\,dx$ is convergent.

(4) Check the convergence of $\int_0^3\frac{dx}{(x-1)}dx$ convergent?

(5) Check convergence of $\int_0^5\frac{sin3xdx}{x^{1/2}}$.

(6) Prove that $B(m,m)B\left(m+\frac{1}{2},m+\frac{1}{2}\right)=\frac{\pi m^{-1}}{2^{4m-1}}$ & $\frac{B(m,n+1)}{n}=\frac{B(m+1,n)}{m}=\frac{B(m,n)}{m+n}$

(7)Prove that $\int_0^\infty\frac{x^8(1-x^6)}{(1+x)^{24}}dx=0.$

(8) Evaluate following:

(i) $\int_0^\infty x^me^{-ax^n}dx$ (ii) $\int_0^1 x^4cos^{-1}x\,dx$.

(9) Express $\int_0^1 x^m(1-x^n)^p dx$ in terms of Gamma Function.

(10) Find the volume of right circular cone of base r and height h.

(11) Find the area of surface generated by revolving the cardioid $r=a(1+cos\theta)$ about the initial line.

(12) Find the perimeter of the loop of the curve $3ay^2=x(x-a)^2, a>0.$

(13) Find the area of the surface by revolving the graph of the curve $y=x^3\ from$

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$x = 0\ to\ x = 1$ about the x-axis.

Assignment-2

Subject:-Mathematics-1 Topic:- Convergence of Sequence and Series.

Weightage: 20%

5. **Test the convergence of the following series**

(1) $4 - 1 + \frac{1}{4} - \frac{1}{16} + \cdots$ (2) $\sum_{n=1}^{\infty} \frac{6}{(2n-1)(2n+1)}$ (3) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}(\sqrt{n}+1)}$

(4) $\sum_{n=1}^{\infty} \frac{8tan^{-1}(n)}{1+n^2}$ (5) $\sum_{n=1}^{\infty} \frac{1}{1^2+2^2+3^2+\cdots+n^2}$ (6) $\sum_{n=1}^{\infty} \left(\frac{n}{3n+1}\right)^n$ (7) $\sum_{n=1}^{\infty} \frac{(n+1)(n+2)}{n!}$

(8) $\sum_{n=1}^{\infty} \frac{n}{(n^3+1)^{\frac{3}{7}}}$ (9) $\sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n^2}\right)$ (10) $\sum_{n=2}^{\infty} \frac{(-1)^{n+1}}{\ln n}$ (11) $\sum_{n=1}^{\infty} \frac{\cos n\pi}{n}$

(12) $\sum_{n=1}^{\infty} (-1)^n \cdot n^2 \cdot \left(\frac{2}{3}\right)^n$

6. **For what values of x does the following series converge? And find their radius of convergence.**

(1) $1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$ (2) $\sum_{n=1}^{\infty} \frac{(-1)^n \cdot (x)^n}{\sqrt{n^2+3}}$ (3) $\sum_{n=1}^{\infty} \sqrt[n]{n} \cdot (2x+5)^n$

7. **Find Taylor series for following functions at given point [OR in powers of (x-a) .**

(1) $f(x) = lnx; a = 2$ (2) $f(x) = sin2x; a = \frac{\pi}{4}$ (3) $f(x) = ln(x+1)\ ; a = 0$

(4) $3x^3 + 7x^2 - 5x - 4$ at x = 0

8. **Find Maclaurin series for following functions.**

(1) $f(x) = \sinh x$ (2) $f(x) = cos^{-1}x$ (3) $f(x) = e^{-8x}$ (4) $f(x) = cot^{-1}\ x$

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Assignment-3

Subject:-Mathematics-1 Topic: Fourier Series

Weightage: 10%

Ex-1 Find Fourier series of the function of $f(x) = x^2\ , -l \le x \le l\ ,\ p = 2l.$

Ex-2 Find Fourier series of the function of $f(x) = x - x^2\ , -1 \le x \le 1\ ,\ p = 2L = 2.$

Ex-3 Find Fourier series of the function of $f(x) = x,\ (-\pi < x < \pi),\ \ f(x + 2\pi) = f(x).$

Ex-4 Find Fourier series of

(1) $f(x) = |x|\ ; -\pi < x < \pi.$
(2) $f(x) = e^x\ , 0 \le x \le 2\pi$

Ex-5 Find Fourier series of $f(x) = \begin{cases} 0, -2 < x < 0 \\ 1,\ \ 0 < x < 2 \end{cases}.$

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Ex-6 Find Fourier series of the function of $f(x) = \begin{cases} -\pi, & -\pi < x < 0 \\ x, & 0 < x < \pi, \end{cases}$ hence show that

$$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots = \frac{\pi^2}{8}.$$

Ex-7 Find Fourier series of the function of $f(x) = \begin{cases} x, & 0 \le x \le \pi \\ 2\pi - x, & \pi \le x \le 2\pi, \end{cases}$

Ex-8 Find Fourier sine series of the function

(1) $f(x) = e^x;\ 0 < x < \pi.$
(2) $f(x) = 2x;\ 0 < x < 1.$
(3) $f(x) = \pi x - x^2;\ 0 < x < \pi.$

Ex-9 Find Fourier cosine series of the function

(1) $f(x) = e^{-x},\ 0 \le x \le \pi$
(2) $f(x) = x^2;\ 0 < x < \pi.$
(3) $f(x) = \pi - x;\ 0 < x < \pi$

Ex-10 Find cosine series for $f(x) = sinx$ where $0 < x < \pi$, hence deduct that

$$\frac{1}{1} - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots = \frac{\pi}{4}.$$

Assignment-4

Subject:- Mathematics 01 **Topic:- Partial Differentiation**

Weightage: 20%

Ex-1 Find the limits:

(1) $\lim\limits_{\substack{(x,y)\to(1,1)\\ x\neq 1}} \frac{xy-y-2x+2}{x-1}$ (2) $\lim\limits_{(x,y)\to(1,1)} \cos\sqrt[3]{|xy|-1}$

(3) $\lim\limits_{(x,y)\to(0,0)} \frac{e^y \sin x}{x}$ (4) $\lim\limits_{(x,y,z)\to(0,-2,0)} \ln\left(\sqrt{x^2+y^2+z^2}\right)$

Ex-2 Determine whether limit of the following functions are exit or not about $(x, y) \to (0, 0)$? If they exist find the value of the limit.

(1) $f(x,y) = \frac{x^2y^2}{x^4+3y^4}$ (2) $f(x,y) = \frac{-x}{\sqrt{x^2+y^2}}$

Ex-3 Show that $\lim\limits_{(x,y)\to(0,0)} \frac{xy}{\sqrt{x^2+y^2}} = 0.$

Ex-4 Discuss the continuity of $f(x,y) = \begin{cases} \frac{x^3-y^3}{x^2+y^2} & ; \quad (x,y) \neq (0,0) \\ 0 & ; \quad (x,y) = (0,0) \end{cases}.$

Ex-5 If $z = e^{xy}$, prove that $\frac{\partial^2 z}{\partial x\, \partial y} = \frac{\partial^2 z}{\partial y\, \partial x}$.

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Ex-6 If $u = \log(x^2 + y^2 + z^2)$, prove that $x\frac{\partial^2 u}{\partial y\,\partial z} = y\frac{\partial^2 u}{\partial z\,\partial x} = z\frac{\partial^2 u}{\partial x\,\partial y}$.

Ex-7 Calculate f_{xxyz}, if $f(x, y, z) = x^2yz + yz$.

Ex-8 If $u = r^2\cos 2\theta$, show that $\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} = 0$.

Ex-9 If $u = \frac{x^3y^3z^3}{x^3+y^3+z^3}$ then prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 6u$.

Ex-10 If $u = \sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$ prove that (1) $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}\tan u$

(2) $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\,\partial y} + y^2\frac{\partial^2 u}{\partial y^2} = \frac{1}{4}(\tan^3 u - \tan u)$

Ex-11 If $u = f\left(\frac{x}{y}, \frac{y}{z}, \frac{z}{x}\right)$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0$.

Ex-12 Find $\frac{\partial W}{\partial r}$ and $\frac{\partial W}{\partial s}$, if $w = x + 2y + z^2, x = r/s, y = r^2 + \ln s, z = 2r$.

Ex-13 If $z = f(u, v)$ and $u = x^2 - y^2, v = 2xy$, show that $\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 = 4(u^2 + v^2)^{1/2}\left[\left(\frac{\partial z}{\partial u}\right)^2 + \left(\frac{\partial z}{\partial v}\right)^2\right]$.

Ex-14 Find $\frac{du}{dt}$ for following: (1) $u = x^3y + 4xy^3, x = \sin t, y = \cos t$ at $t = 0$ (2) $u = \sin\left(\frac{x}{y}\right), x = e^t, y = t^2$.

Ex-15 Find the value of $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ using partial derivative, $xy + y^2 - 3x - 3 = 0$ at (-1, 1)

Ex-16 Find the gradient of $f(x, y, z) = 2z^3 - 3(y^2 + x^2)z + tan^{-1}(xz)$, at the point (1, 1, 1).

Ex-17 If $r = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$, evaluate the following (1) ∇r^n (2) $\nabla\,|r|^3$ (3) $\nabla\left(3r^2 - 4\sqrt{r} + 6r^{-1/3}\right)$ (4) ∇r (5) $\nabla\,(\ln r)$ (6) $\nabla\,(1/r)$

Ex-18 Find the directional derivative of following functions at given point and direction:

(1) $f(x, y, z) = x^3 - xy^2 - z$ at (1, 1, 0) in the direction $v = 2\hat{\imath} - 3\hat{\jmath} + 6\hat{k}$

(2) $f(x, y, z) = x^2y^2z^2$ at (1, 1, -1) along a direction equally inclined with coordinate axes.

(3) $T(x, y, z) = xy + yz + zx$ at (1, 1, 1) in the direction $v = 3\hat{\imath} - 4\hat{k}$

Ex-19 Find the equation for the tangent plane and normal line at the point P_0 on the given surface

(1) $z^2 - 2x^2 - 2y^2 - 12 = 0$; $P_0(1, -1, 4)$

(2) $\cos\pi x - x^2y + e^{xz} + yz = 4$; $P_0(0,1,2)$ (3) $z = e^{-(x^2+y^2)}$; $P_0(0,0,1)$

Ex-20 Find the local extreme values and saddle points of the following functions.

(1) $f(x, y) = x^2 + xy + y^2 + 3x - 3y + 4$ (3) $f(x, y) = x + y + \frac{1}{x} + \frac{1}{y}$

(2) $f(x, y) = \sin x + \sin y + \sin(x + y)$ (4) $f(x, y) = x^3 + 3xy + y^3$

Ex-21 Find the absolute maxima and minima of the functions on the given domains.

(1) $f(x, y) = 2x^2 - 4x + y^2 - 4y + 1$ on the closed triangular plate bounded by the lines $x = 0, y = 2, y = 2x$ in the first quadrant.

(2) $f(x, y) = 48xy - 32x^3 - 24y^2$ on the rectangular plate $0 \le x \le 1, 0 \le y \le 1$.

Ex-22 The pressure P at any point (x, y, z) in space is $P = 400xyz^2$. Find the highest pressure on the surface of the unit sphere $x^2 + y^2 + z^2 = 1$.

Ex-23 Find the points on the sphere $x^2 + y^2 + z^2 = 4$ that are closest to and farthest from the point (3, 1, -1)

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Assignment-5

Subject:- MATHEMATICS-1 **Topic:- MULTIPLE INTEGRALS.**

Weightage: 20%

1. Sketch the region of integration and evaluate the following integrals.

(a) $\int_0^3 \int_{-2}^0 (x^2y - 2xy)dydx$ (b) $\int_{\pi}^{2\pi} \int_0^{\pi} (\sin x + \cos y)dxdy$ (c) $\int_1^2 \int_y^{y^2} dxdy$

2. Integrate $f(x, y) = \frac{1}{xy}$ over the square $1 \leq x \leq 2,\ 1 \leq y \leq 2$.

3. Integrate $f(s, t) = e^s \cdot \ln t$ over the region in the first quadrant of the ST-plane that lies above the curve $s = \ln t$ from t=1 to t=2.

4. Sketch the region of integration and evaluate the following integrals by reversing its order.

(a) $\int_0^a \int_y^a \frac{x}{x^2+y^2} dxdy$ (b) $\int_0^8 \int_{\sqrt[3]{x}}^2 \frac{dxdy}{y^4+1}$ (c) $\int_0^2 \int_0^{4-x^2} \frac{x \cdot e^{2y}}{4-y} dxdy$

5. Evaluate $\iint_R xy\ dA$, where R is the region bounded by the lines y=0, x=0 and x+y=2.

6. Find the volume of the solid that is bounded by the paraboloid $z = x^2 + y^2$ and below by the region enclosed by the lines y=x, x=0 and x+y=2.

7. Find the volume of the solid cut from the first octant by the surface $= 4 - x^2 - y$.

8. Sketch the region bounded by the given lines and curves then express the region's area as a double integral and evaluate the integral.

(a) The co-ordinate axis and the line x+y=2 (b) Parabolas $x=y-y^2$ and $x=-y$.

9. Evaluate the following integrals.

(a) $\int_0^e \int_0^e \int_0^e \frac{1}{xyz} dzdydx$ (b) $\int_{-1}^1 \int_{-1}^1 \int_{-1}^1 x + y + z\ dzdydx$ (c) $\int_0^2 \int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} \int_0^{2x+y} dzdxdy$

10. Change the Cartesian integral into the polar integral then evaluate the polar integral.

(a) $\int_0^2 \int_0^{\sqrt{4-y^2}} (x^2 + y^2)dxdy$ (b) $\int_0^2 \int_0^{\sqrt{1-(x-1)^2}} \frac{x+y}{x^2+y^2} dydx$ (c) $\int_{-1}^1 \int_{-\sqrt{1-y^2}}^0 \frac{4\sqrt{x^2+y^2}}{1+x^2+y^2} dxdy$

11. Solve the system $u = 2y + x, v = x - y$ for x and y in terms of u and v then find value of the Jacobian $\frac{\partial(x,y)}{\partial(u,v)}$.

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Assignment – 6

Subject:- MATHEMATICS-1 **Topic:-Linear algebra.**

Weightage: 15%

Ex-1 Define following with examples.

(1)Matrix (2)Row matrix (3)Column matrix (4)Zero matrix (5)Square matrix (6)Identity matrix (7)Lower Triangular matrix (8)Upper Triangular matrix (9)Triangular matrix (10)Diagonal matrix (11)Symmetric matrix (12)Skew- Symmetric matrix (13)Trace of a matrix (14)Transpose of a matrix (15)Orthogonal matrix (16)Equal matrices (17)Addition of two matrices (18)Subtraction of two matrices (19)Scalar multiplication of a matrix (20)Product of two matrices (21)Determinant of a matrix

Ex-2 Using Gauss-Jordan method find A^{-1} if

(i) $A=\begin{bmatrix} 3 & 4 & 6 \\ 6 & 7 & 8 \\ 9 & 10 & 11 \end{bmatrix}$ (ii) $A=\begin{bmatrix} 3 & -2 & 0 & -1 \\ 0 & 2 & 2 & 1 \\ 1 & -2 & -3 & -2 \\ 0 & 1 & 2 & 1 \end{bmatrix}$

Ex-3 Find the rank using row echelon form.

(i) $\begin{bmatrix} 1 & 5 & 9 \\ 4 & 8 & 12 \\ 7 & 11 & 15 \end{bmatrix}$ (ii) $\begin{bmatrix} 2 & -2 & 0 & 6 \\ 4 & 2 & 0 & 2 \\ 1 & -1 & 0 & 3 \\ 1 & -2 & 1 & 2 \end{bmatrix}$

EX-4 Solve following system of equation using Gauss-elimination method.

(1)$x+y=3, 2x-y+2z=-2, 3x+y=3$

(2)$3x-4y=10, -5x+8y=-17, -3x+12y=-12$

Ex-5 Solve each of following system by Gauss-Jordan elimination Method.

(1)$2x-y+3z=3, 4x+2y-2z=2, 6x-3y+z=9$

Ex-6 Investigate the values of a and b so equation

$2x+3y+5z=9, 7x+3y-2z=8, 2x+3y+az=b$ have

(i) No Solution (ii) Unique Solution (iii) Infinite number of solutions.

Ex-7 Find the Eigen values and Eigen vectors of

(1) $\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$ (2) $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ (3) $\begin{bmatrix} 11 & -4 & -7 \\ 7 & -2 & -5 \\ 10 & -4 & -6 \end{bmatrix}$

Ex-8 Using Cayley - Hamilton , find A^{-1}, $A=\begin{bmatrix} 7 & -1 & 3 \\ 6 & 1 & 4 \\ 2 & 4 & 8 \end{bmatrix}$

Ex-9 If $A=\begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$ then by using cayley -Hamilton theorem prove that,

$$A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I = \begin{bmatrix} 8 & 5 & 5 \\ 0 & 3 & 0 \\ 5 & 5 & 8 \end{bmatrix}$$

Ex-10 Find a matrix P that diagonalize the matrix $A=\begin{bmatrix} 1 & 0 \\ 6 & -1 \end{bmatrix}$ & determine $P^{-1}AP$.

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Ex-11 Find a matrix P that diagonalize the matrix A, and determine $P^{-1}AP$.

(a)$A = \begin{bmatrix} 1 & 0 \\ 6 & -1 \end{bmatrix}$ (b) $A = \begin{bmatrix} 9 & -1 & 9 \\ 3 & -1 & 3 \\ -7 & 1 & -7 \end{bmatrix}$

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